Name of the Course : B.Sc. (Hons.) Mathematics CBCS (LOCF)

Unique Paper Code : 32351303

Name of the Paper : BMATH307 – Multivariate Calculus

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Attempt any four questions. All questions carry equal marks.

1. Let $f(x,y) = \begin{cases} \frac{y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Is the function f continuous at $(0,0)$? Justify your answer.

Find an equation for the tangent plane to the surface $z = f(x,y)$ defined above at the point $P_0\left(1, 2, \frac{8}{5}\right)$.

Also find the directional derivative of $f(x,y)$ at $P_0(1,2)$ in the direction of $\mathbf{v} = 3\boldsymbol{i} - 2\boldsymbol{j}$.

2. Find the critical point and classify each point as a relative minimum, relative maximum, or a saddle point of

$$f(x,y) = xye^{-8(x^2+y^2)}.$$

Find the maximum and minimum values of $f(x,y,z) = xyz$ subject to the constraint

$$x^2 + 2y^2 + 4z^2 = 24.$$

Where is the function $f(x,y) = \sqrt{x^2+y^2}$ differentiable?

3. Compute $\iint_R xe^{xy}dA$ where R is the rectangle $0 \le x \le 1, 1 \le y \le 2$, using iterated integrals in both orders.

Evaluate $\iint_R 6x^2y\, dA$ if R is the region bounded between the curves $y = x, y = 1$ and $4y = x^2$.

Find the area of the region bounded between the curves $r_1(\theta) = 2 + \sin 3\theta$ and $r_2(\theta) = 4 - \cos 3\theta$.

4. Find the mass of the ellipsoid $4x^2 + 4y^2 + z^2 = 16$ lying above the xy-plane if the density is given by $\delta(x,y,z) = z$.

Determine the centroid of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the xy-plane plane where the density is given by $\delta(x,y,z) = z$.

Compute $\int_0^1 \int_0^1 x^2 y \, dx \, dy$ by changing $u = x$ and $v = xy$.

5. Evaluate $\oint_C (x^2 z dx - yx^2 dy + 3dz)$ where C is the boundary of the triangle with vertices $(0, 0, 0), (1, 1, 0)$ and $(1, 1, 1)$.

Find a non-zero function h for which

$$\boldsymbol{F}(x, y) = h(x)(x \sin y + y \cos y)\boldsymbol{i} + h(x)(x \cos y - y \sin y)\boldsymbol{j}$$

is conservative.

Using line integral, find the area of the region enclosed by the asteroid

$$x = a \cos^3 t, \;\; y = a \sin^3 t \; (0 \le t \le 2\pi).$$

6. Find the mass of the lamina that is the portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 2$ with constant density δ_0.

Verify Stokes' Theorem if $\boldsymbol{F}(x, y, z) = (x - y)\boldsymbol{i} + (y - z)\boldsymbol{j} + (z - x)\boldsymbol{k}$ and S be the portion of the plane $x + y + z = 1$ in the first octant assuming that the surface has an upward orientation.

Using the Divergence Theorem, evaluate $\iint_S \boldsymbol{F}.\boldsymbol{N} dS$, where $\boldsymbol{F}(x, y, z) = (z^3\boldsymbol{i} - x^3\boldsymbol{j} + y^3\boldsymbol{k})$ and S is the sphere $x^2 + y^2 + z^2 = a^2$, with outward unit normal vector $\boldsymbol{N}$.